J. Fluid Mech. (2008), *vol.* 603, *pp.* 474–475.
Printed in the United Kingdom

BOOK REVIEW

Computational Methods for Multiphase Flow. By A. Prosperetti & G. Tryggvason. Cambridge University Press, 2007. 470 pp. ISBN 13 978 0521 84764 3. £55 (hardback).

J. Fluid Mech. (2008), *vol.* 603, doi:10.1017/S0022112008001250

Computational Fluid Dynamics (CFD) is becoming an increasingly important branch of fluid mechanics. Practitioners are now quite confident in using numerical simulation to predict low-Reynolds-number flows of a single phase. One of the big challenges for the CFD community is to predict high-Reynolds-number flows with turbulence, where a revolutionary breakthrough will be necessary. However, there is another important area of research in which CFD has demonstrated convincingly its considerable power. Multiphase flow is rich in physics and important in industrial applications. The last few decades have seen an explosion of techniques for the computational methods for multiphase flow and dramatic progress has been made. There is an urgent need for a book surveying recent achievements and showing the future direction of research and applications. This book aims to fulfil this need. To some extent, CFD is not an exact science for which it is sufficient to study the theory in abstraction; it is an art, which one has to practice oneself to fully appreciate the subtlety of each method. This book was written by world-leading experts with much first-hand experience, which makes the book even more valuable.

The book is composed of two parts. The first part is on direct simulation and the second on simulation with modelling. Directed by my experience, I will concentrate my comments on the first part. Methods for direct simulations are divided into two main categories: methods on fixed meshes and methods with moving meshes. The first is the so-called ‘one-fluid’ approach. This includes the front tracking method, the volume of fluid (VOF) method and the level-set method. It is very well summarized in chapter 2 and chapter 3, which are written in concise and clear style, and easy to read. All important issues are adequately discussed: the numerical (projection) method of Navier–Stokes equations; the appropriate boundary conditions; the modelling of the surface tension on the interface, and the related numerical difficulty of parasitic currents; the advantage of the harmonic average of viscosity over the simple arithmetic average; the effect of topological changes in the interface (confidence in the numerical treatment of thread breakup and uncertainty of interface reconnection). The authors also provide several benchmark tests, one of which is the oscillation frequency of a circular or spherical drop. This is the classic normal-mode analysis of Lamb (1932). A similar test could be the solution obtained by Prosperetti (1980) expressed as a Laplace transform. This is an initial-value problem perturbing only the interface shape but not the velocity field, and hence is convenient for a numerical set-up.

It is easy to prove that the harmonic average of viscosity gives the exact solution of the two-layer Couette flow configuration (Coward *et al.* 1997). This is, however, not true in general. Indeed, the resolution of the boundary condition on the interface is the weak point of the ‘one-fluid’ approach. The situation can be remedied by using moving meshes, where the interface is aligned with mesh lines in two dimensions and faces in three dimensions. The boundary condition can then be resolved neatly and accurately using the finite volume method (Li *et al.* 2005) and the finite element method (FEM). Chapter 5 presents all the necessary ingredients for the arbitrary Lagrangian–Eulerian (ALE) method based on FEM: a good choice of elements (Taylor element: P2 for velocity and P1 for pressure), the adaptive mesh strategy and the GMRES method for the solution of discretized systems. It also includes a new feature: the fluid can be a viscoelastic liquid; the solution of viscoelastic liquid flow is a formidable task. Although this chapter only treats particulate flows, it can be easily extended to other types of multiphase flows. In a recent paper (Dai & Schmidt 2005) mesh reconnection algorithms were used to keep the mesh quality, instead of remeshing from scratch as suggested here. The inconveniences of FEM are the complicated book-keeping of data structure and the computation cost. It is difficult to deal with topological changes of interfaces. A skilled programmer should be able to write a workable 'one-fluid' approach code after reading chapter 2 and chapter 3. The task of writing a FEM code is much more difficult.

Another useful method for multiphase flow computation is the boundary integral method summarized in chapter 7. Its main advantage is that the implementation involves only integration on the interface, therefore allowing high resolutions, especially in three-dimensional simulations. However, the underlying mathematical theory is difficult. The researcher who wishes to use the 'one-fluid' approach or the FEM method can start immediately from the familiar Navier–Stokes equation. Practitioners of boundary integral methods have to first work rather hard to obtain the governing equations! This chapter uses more than two thirds of its length in order to obtain equation (7.72) or (7.86). The rest is dedicated to numerical techniques for solving these two equations. These techniques include surface discretization and approximation, evaluation of surface integrals and iterative solution methods. They are well presented. The most difficult numerical issue is the singularity of the Green's kernels. However, once the above difficulties are overcome, the effort is worth the reward. The boundary integral method currently produces perhaps the best accuracy and the finest details of multiphase flow with zero Reynolds number, as we can see from two examples reproduced here. Finally, the boundary integral methods can only be applied to Stokes flow and inviscid flow, thus excluding all interesting and important finite-Reynolds-number flows.

This book covers almost all currently used computational methods for multiphase flow. Its primary propose is 'to enable the reader to understand the current literature' on multiphase flow and 'to provide a solid basis on which to build both applications and research'. Its goal is undoubtedly well fulfilled. I highly recommend this volume to those readers who wish acquaint themselves with computational methods for multiphase flow.

REFERENCES

Coward, A. V., Renardy, Y., Renardy, M. & Richards, J. R. 1997 Temporal evolution of periodic disturbances in two-layer Couette flow. *J. Comput. Phys.* **132**, 346–361.

Dai, M. & Schmidt, D. 2005 Adaptive tetrahedral meshing in free-surface flow. *J. Comput. Phys.* **208**, 228–252.

Jie L., J., Hesse, M., Ziegler, J. & Woods, A. W. 2005 An arbitrary Lagrangian Eulerian method for moving-boundary problems and its application to jumping over water. *J. Comput. Phys.* **208**, 289–314.

Lamb, H. 1932 *Hydrodynamics.* Cambridge University Press.

Prosperetti, A. 1980 Free oscillations of drops and bubbles: the initial-value problem. *J. Fluid Mech.* **100**, 333–347.

Jie Li